00:00:05.430 --> 00:00:07.838
So today we're going to talk about

00:00:07.838 --> 00:00:09.983
factoring and we're going to talk

00:00:09.983 --> 00:00:12.065
about a specific type of factoring,

00:00:12.070 --> 00:00:15.070
so we're gonna look at factoring.

00:00:15.070 --> 00:00:18.437
And we're going to factor quadratics where

00:00:18.437 --> 00:00:20.909
the leading coefficient is not a one.

00:00:20.910 --> 00:00:23.724
So if you remember what a typical

00:00:23.724 --> 00:00:26.103
quadratic looks like a generic one,

00:00:26.103 --> 00:00:29.210
it's a X ^2 plus BX plus C,

00:00:29.210 --> 00:00:32.590
and so we're looking at ones where the

00:00:32.590 --> 00:00:35.999

A that's the leading coefficient is not.

00:00:36.000 --> 00:00:39.222

Equal to 1. And a lot of times when

00:00:39.222 --> 00:00:41.967

you're taught how to factor these,

00:00:41.970 --> 00:00:44.190

you're taught the AC method,

00:00:44.190 --> 00:00:45.906

or to just guess and check.

00:00:45.910 --> 00:00:47.416

But I actually have a different

00:00:47.416 --> 00:00:48.909

method called the bottoms up method,

00:00:48.910 --> 00:00:49.966

and I learned this.

00:00:49.966 --> 00:00:51.955

Actually, I was teaching and a

00:00:51.955 --> 00:00:53.940

student actually suggested it to me.

00:00:53.940 --> 00:00:56.404
They had learned it a long time ago

00:00:56.404 --> 00:00:59.470
and I was looked into it and they it

00:00:59.470 --> 00:01:01.630
was really actually easier than the

00:01:01.630 --> 00:01:04.049
AC method and the guests in check.

00:01:04.050 --> 00:01:05.890
And I've been teaching it to my students

00:01:05.890 --> 00:01:07.680
ever since, and they all love it.

00:01:07.680 --> 00:01:08.970
So every time I teach it,

00:01:08.970 --> 00:01:09.310
they're like,

00:01:09.310 --> 00:01:10.980
why did no one show this to me sooner?

00:01:10.980 --> 00:01:14.796
So we're going to work through an example.

00:01:14.800 --> 00:01:16.802
There's one thing you want to be

00:01:16.802 --> 00:01:18.617
careful of when you do the gap.

00:01:18.620 --> 00:01:20.818
When you do the bottoms up method.

00:01:20.820 --> 00:01:22.808
So this is the bottoms up method.

00:01:26.100 --> 00:01:27.558
And the first thing you have

00:01:27.558 --> 00:01:29.302
to be careful of when you do

00:01:29.302 --> 00:01:30.730
the bottoms up method is that

00:01:30.730 --> 00:01:32.318
if there's a common factor,

00:01:32.320 --> 00:01:33.940
you must factor it out first.

00:01:33.940 --> 00:01:35.704

The bottoms up method will not

00:01:35.704 --> 00:01:37.676

work if you don't factor out

00:01:37.676 --> 00:01:40.508

that common factor, so you must.

00:01:42.560 --> 00:01:47.730

Factor out. Any? Common.

00:01:50.380 --> 00:01:51.280

Factors.

00:01:53.580 --> 00:01:55.724

So the first example I'm going to do

00:01:55.724 --> 00:01:58.357

doesn't have any common factors to it, so.

00:01:58.357 --> 00:02:02.359

For example, we want to factor.

00:02:04.870 --> 00:02:10.139

15 X squared minus seven X - 2

00:02:10.139 --> 00:02:13.331

and you can see that the 15 the

00:02:13.331 --> 00:02:16.300
minus seven and the minus two.

00:02:16.300 --> 00:02:17.492
They don't have anything

00:02:17.492 --> 00:02:19.280
in common and the 15 here.

00:02:19.280 --> 00:02:21.105
The coefficient that's in front

00:02:21.105 --> 00:02:23.758
of the X squared term is not one,

00:02:23.760 --> 00:02:25.308
so this is a great candidate

00:02:25.308 --> 00:02:26.790
for the bottoms up method.

00:02:26.790 --> 00:02:27.834
The first thing we're going to

00:02:27.834 --> 00:02:28.940
do on the bottoms up method,

00:02:28.940 --> 00:02:31.096
it's very similar to the AC method

00:02:31.096 --> 00:02:33.538
and that is we're going to multiply

00:02:33.538 --> 00:02:35.710
this 15 times this negative 2.

00:02:35.710 --> 00:02:38.818
So I'm factoring this is kind of

00:02:38.818 --> 00:02:41.849
like iQiyi and lucky looks ugly,

00:02:41.850 --> 00:02:43.554
so instead we're going to factor

00:02:43.554 --> 00:02:44.406
a related problem.

00:02:44.410 --> 00:02:45.838
So the related problem we're going

00:02:45.838 --> 00:02:47.604
to get is we're going to multiply

00:02:47.604 --> 00:02:49.128
the 15 times the negative two,

00:02:49.130 --> 00:02:51.391
and so we're going to look at

00:02:51.391 --> 00:02:54.670
factoring X ^2 - 7 X and then 15

00:02:54.670 --> 00:02:57.638
times negative two is minus 30, right?

00:02:57.638 --> 00:03:01.102
So I did the 15 times negative 2 and

00:03:01.102 --> 00:03:03.550
that's the same first step in the AC method.

00:03:03.550 --> 00:03:05.286
So if you're used to the AC method,

00:03:05.290 --> 00:03:07.360
it's the same first step.

00:03:07.360 --> 00:03:09.172
But this is a much nicer

00:03:09.172 --> 00:03:10.450
polynomial to factor, right?

00:03:10.450 --> 00:03:13.040

The leading coefficient is now a wine,

00:03:13.040 --> 00:03:14.640
so that's what we're going to start with.

00:03:14.640 --> 00:03:17.346
So I'm looking for things that

00:03:17.346 --> 00:03:20.033
multiply to give me negative 30

00:03:20.033 --> 00:03:23.201
and add to give me a negative 7.

00:03:23.210 --> 00:03:25.863
So if I start thinking about things

00:03:25.863 --> 00:03:28.540
that multiply to give me negative 30,

00:03:28.540 --> 00:03:31.816
I could look at, say, two and negative 15.

00:03:31.816 --> 00:03:36.119
But if I add two and negative 15,

00:03:36.120 --> 00:03:39.256
I get negative 13. So that doesn't work.

00:03:39.260 --> 00:03:41.852
Another thing I could try is maybe I

00:03:41.852 --> 00:03:44.741
try a negative three and a positive

00:03:44.741 --> 00:03:47.370
10 so negative three times positive

00:03:47.370 --> 00:03:50.458
10 does in fact give me negative 30,

00:03:50.460 --> 00:03:53.078
but negative 3 + a positive 10

00:03:53.078 --> 00:03:55.299
gives me a positive 7.

00:03:55.300 --> 00:03:56.976
So I'm really close,

00:03:56.976 --> 00:03:59.490
but I'm not quite there yet.

00:03:59.490 --> 00:04:02.138
So the only thing difference is I want

00:04:02.138 --> 00:04:04.557
a negative 7 and I got a positive 7.

00:04:04.560 --> 00:04:06.582
So if you're in this situation

00:04:06.582 --> 00:04:07.593
you're so close.

00:04:07.600 --> 00:04:09.490
All you have to do is flip flop the

00:04:09.490 --> 00:04:11.293
signs here so we're instead going to

00:04:11.293 --> 00:04:13.300
do a positive three and a negative 10.

00:04:13.300 --> 00:04:14.806
So I've got this positive three.

00:04:14.810 --> 00:04:15.935
It's negative 10.

00:04:15.935 --> 00:04:19.539
If I add them together I get negative 7.

00:04:19.540 --> 00:04:21.820
OK, so.

00:04:21.820 --> 00:04:23.980

Just like if you were factoring

00:04:23.980 --> 00:04:24.700
this polynomial,

00:04:24.700 --> 00:04:26.652
this quadratic like before,

00:04:26.652 --> 00:04:30.198
this is going to factor into X + 3.

00:04:30.200 --> 00:04:34.445
And X - 10 and now here is where

00:04:34.445 --> 00:04:36.200
the bottoms up method comes in.

00:04:36.200 --> 00:04:39.000
So we're going to take the leading

00:04:39.000 --> 00:04:40.250
coefficient, the 15,

00:04:40.250 --> 00:04:42.800
and we're going to divide these

00:04:42.800 --> 00:04:44.532
two numbers by 15 now.

00:04:44.532 --> 00:04:46.392
A lot of mathematicians don't

00:04:46.392 --> 00:04:47.880
always like this method,

00:04:47.880 --> 00:04:49.259
'cause it's a little gimmicky and you

00:04:49.259 --> 00:04:50.679
kind of just have to remember it,

00:04:50.680 --> 00:04:52.660
but it works all the time.

00:04:52.660 --> 00:04:54.190
So while a lot of mathematicians

00:04:54.190 --> 00:04:54.955
don't like it,

00:04:54.960 --> 00:04:56.626
a lot of students love it because

00:04:56.626 --> 00:04:58.240
it takes away that guessing and

00:04:58.240 --> 00:05:00.221
checking and frustration that a lot of

00:05:00.277 --> 00:05:01.737
people get trying to just

00:05:01.737 --> 00:05:02.905
factor from the beginning.

00:05:02.910 --> 00:05:05.115
So I divide it by the bottom and the

00:05:05.115 --> 00:05:07.193
reason it's called the bottoms up method

00:05:07.193 --> 00:05:09.325
is because at this point we're going

00:05:09.325 --> 00:05:11.752
to look at each of these fractions so I

00:05:11.752 --> 00:05:15.021
have this 3 / 15 and this minus 10 / 15.

00:05:15.021 --> 00:05:17.548
And I want to see whether or

00:05:17.548 --> 00:05:19.670
not there's a common factor.

00:05:19.670 --> 00:05:21.846

So if you think about 3 in 15,

00:05:21.850 --> 00:05:23.926
is there any number that goes

00:05:23.926 --> 00:05:26.300
into three and also goes into 15?

00:05:26.300 --> 00:05:28.400
What do we think?

00:05:28.400 --> 00:05:29.940
Hopefully you said three.

00:05:29.940 --> 00:05:31.865
So three goes into three.

00:05:31.870 --> 00:05:36.390
One time, three goes into 15 five times,

00:05:36.390 --> 00:05:39.004
and over here I have 10 / 15.

00:05:39.004 --> 00:05:41.650
So again, let's try to think of a number

00:05:41.725 --> 00:05:44.069
that goes into 10 and goes into 15.

00:05:44.070 --> 00:05:47.868

If there is one and there is right five.

00:05:47.870 --> 00:05:52.830

So five goes into Ten 2 * 5 goes into 15.

00:05:52.830 --> 00:05:53.850

Three times.

00:05:53.850 --> 00:05:58.316

OK, so I've took my coefficient here.

00:05:58.320 --> 00:06:00.360

I divided it on the bottoms and then

00:06:00.360 --> 00:06:02.317

I reduced and again the reason it's

00:06:02.317 --> 00:06:04.039

called the bottoms up is because

00:06:04.039 --> 00:06:05.943

now this bottom number if I'm left

00:06:05.943 --> 00:06:08.210

with a bottom number it's going

00:06:08.210 --> 00:06:10.648

to come up so this bottom number

00:06:10.648 --> 00:06:13.045

is going to come up in front and

00:06:13.045 --> 00:06:14.857

so this is going to factor.

00:06:14.860 --> 00:06:17.340

Give us the factor of our original problem.

00:06:17.340 --> 00:06:20.546

So the answer to our original problem

00:06:20.546 --> 00:06:23.848

is going to be 5X plus one right?

00:06:23.848 --> 00:06:27.827

So I took the five and I moved it up.

00:06:27.830 --> 00:06:31.780

And then I kept the 1 * 3 which

00:06:31.780 --> 00:06:37.354

I moved up and then three X - 2.

00:06:37.354 --> 00:06:39.574

And that's the actual answer

00:06:39.574 --> 00:06:41.350

to our original problem.

00:06:41.350 --> 00:06:43.966
So like I said, it's a little gimmicky.

00:06:43.970 --> 00:06:45.770
It takes a little bit of time to get used to,

00:06:45.770 --> 00:06:47.555
but once you get used to it,

00:06:47.560 --> 00:06:49.912
I found that students really appreciate

00:06:49.912 --> 00:06:52.134
and like this way of factoring.

00:06:52.134 --> 00:06:54.222
And we're going to do another

00:06:54.222 --> 00:06:55.718
example here in a minute.

00:06:59.710 --> 00:07:02.111
OK, so we're going to do another

00:07:02.111 --> 00:07:04.449
example using the bottoms up method,

00:07:04.450 --> 00:07:08.598

so we're gonna factor.

00:07:08.600 --> 00:07:16.020

The following six X ^2 + 9 X Plus three,

00:07:16.020 --> 00:07:19.124

so if you remember when I talked about

00:07:19.124 --> 00:07:22.077

the bottoms up method at the beginning,

00:07:22.080 --> 00:07:23.694

I said if there's anything in

00:07:23.694 --> 00:07:25.619

common you have to factor it out.

00:07:25.620 --> 00:07:29.165

So if you look here 69 and three

00:07:29.165 --> 00:07:31.655

they all have a common factor.

00:07:31.660 --> 00:07:34.036

I could pull a three out of all those

00:07:34.036 --> 00:07:35.155

numbers, so that's the first thing

00:07:35.155 --> 00:07:36.519

I'm going to do to factor this.

00:07:36.520 --> 00:07:38.115

So remember, you're just factoring

00:07:38.115 --> 00:07:39.710

out the greatest common factor.

00:07:39.710 --> 00:07:41.285

But it's really important that

00:07:41.285 --> 00:07:43.250

you do it ahead of time.

00:07:43.250 --> 00:07:45.050

If you're going to use the bottoms up method,

00:07:45.050 --> 00:07:47.087

the bottoms up method will not work.

00:07:47.090 --> 00:07:50.600

If you forget to pull out that common factor,

00:07:50.600 --> 00:07:52.364

so this is going to leave

00:07:52.364 --> 00:07:56.188

us with two X ^2 + 3 X.

00:07:56.188 --> 00:07:57.800

Plus one.

00:07:57.800 --> 00:08:00.565

OK, so I factored out the common

00:08:00.565 --> 00:08:03.063

factor I'm looking at what I have

00:08:03.063 --> 00:08:05.390

left and what I have left here.

00:08:05.390 --> 00:08:07.180

Still has a leading coefficient

00:08:07.180 --> 00:08:09.426

that's not one so great candidate

00:08:09.426 --> 00:08:11.376

for the bottoms up method.

00:08:11.380 --> 00:08:13.516

Remember for the bottoms up method,

00:08:13.520 --> 00:08:16.248

we're going to look at solving a related

00:08:16.248 --> 00:08:18.373

factoring of related problem and

00:08:18.373 --> 00:08:21.115

remember the related problem comes from

00:08:21.115 --> 00:08:23.540

multiplying the A times the see just go,

00:08:23.540 --> 00:08:25.756

put the arc underneath this time since

00:08:25.756 --> 00:08:27.820

I don't have a lot of room up above it.

00:08:27.820 --> 00:08:30.013

So if we multiply 2 * 1,

00:08:30.013 --> 00:08:31.028

we're going to get two,

00:08:31.030 --> 00:08:34.126

so we're going to look at

00:08:34.126 --> 00:08:36.810

factoring X ^2 + 3 X.

00:08:36.810 --> 00:08:40.757

+2 so nice little problem here to factor.

00:08:40.760 --> 00:08:43.286
We're looking for things that multiply

00:08:43.286 --> 00:08:47.460
to give us 2 and add to give us three.

00:08:47.460 --> 00:08:49.172
And if you start to think of the

00:08:49.172 --> 00:08:50.726
things that multiply to give us 2,

00:08:50.730 --> 00:08:51.906
there's not a lot of things.

00:08:51.910 --> 00:08:55.330
There's actually only one thing 1 * 2.

00:08:55.330 --> 00:08:57.898
And luckily 1 * 2 or one and

00:08:57.898 --> 00:09:00.470
two if I add them together,

00:09:00.470 --> 00:09:02.710
do in fact give me a three.

00:09:02.710 --> 00:09:08.098

So this is going to break down into X + 1.

00:09:08.100 --> 00:09:10.660

Times X + 2 right so it's just

00:09:10.660 --> 00:09:12.243

factoring that quadratic with

00:09:12.243 --> 00:09:14.853

that leading coefficient of a one,

00:09:14.860 --> 00:09:16.995

and now we get to do that

00:09:16.995 --> 00:09:18.440

bottoms up part right?

00:09:18.440 --> 00:09:21.016

The thing we want to remember is we're

00:09:21.016 --> 00:09:23.279

trying to factor this polynomial.

00:09:23.280 --> 00:09:26.689

This quadratic here and in this quadratic

00:09:26.689 --> 00:09:28.889

that leading coefficient is a two,

00:09:28.890 --> 00:09:30.666
so I'm not going back to

00:09:30.666 --> 00:09:31.554
the original problem.

00:09:31.560 --> 00:09:33.912
I'm looking at the thing that I'm actually

00:09:33.912 --> 00:09:35.979
trying to factor so this quadratic,

00:09:35.980 --> 00:09:37.380
which is what I'm trying to factor,

00:09:37.380 --> 00:09:38.704
has a leading coefficient.

00:09:38.704 --> 00:09:39.366
Over 2,

00:09:39.370 --> 00:09:43.003
so I'm going to divide each of

00:09:43.003 --> 00:09:46.028
these pieces by two by that.

00:09:46.030 --> 00:09:47.930
Leading coefficient and then just

00:09:47.930 --> 00:09:50.573

like before I'm going to see if

00:09:50.573 --> 00:09:52.258

I can reduce those fractions.

00:09:52.260 --> 00:09:54.644

So if you look at this first fraction,

00:09:54.650 --> 00:09:57.128

the 1 / 2 one half,

00:09:57.130 --> 00:09:58.750

there's nothing to reduce there.

00:09:58.750 --> 00:10:00.010

It's already reduced,

00:10:00.010 --> 00:10:02.950

so we're just gonna leave it alone.

00:10:02.950 --> 00:10:05.380

This 2 / 2 right?

00:10:05.380 --> 00:10:06.584

That actually can be

00:10:06.584 --> 00:10:08.089

reduced down to one right?

00:10:08.090 --> 00:10:10.330
So this is just one and I'm actually

00:10:10.330 --> 00:10:12.759
not even left with a fraction anymore,

00:10:12.760 --> 00:10:14.568
so this is a little different than the

00:10:14.568 --> 00:10:16.735
last one, but it still works the same way.

00:10:16.740 --> 00:10:18.637
So the bottoms up method says if

00:10:18.637 --> 00:10:20.764
you have a bottom, you bring it up

00:10:20.764 --> 00:10:22.480
when you go to write the original.

00:10:22.480 --> 00:10:26.204
The factoring, so we're going to write

00:10:26.204 --> 00:10:28.820
this final factorization this three.

00:10:28.820 --> 00:10:30.437
It just stays there the whole time,

00:10:30.440 --> 00:10:31.358
so it's got to stay there,

00:10:31.360 --> 00:10:32.824
and our answer kind of comes

00:10:32.824 --> 00:10:33.800
along for the ride.

00:10:33.800 --> 00:10:35.977
I pulled it out at the beginning

00:10:35.977 --> 00:10:37.868
and nothing else happens to it.

00:10:37.870 --> 00:10:40.360
And then to this inside part

00:10:40.360 --> 00:10:42.760
here is going to factor.

00:10:42.760 --> 00:10:45.826
This two is going to come up,

00:10:45.830 --> 00:10:46.976
so we're going to have two.

00:10:46.980 --> 00:10:48.751

X and then the number that's left

00:10:48.751 --> 00:10:52.068

here is the one, so plus one.

00:10:52.070 --> 00:10:54.690

And then in this part I don't have a bottom,

00:10:54.690 --> 00:10:56.346

so there's nothing to bring up,

00:10:56.350 --> 00:10:59.246

so this factor is just X + 1,

00:10:59.250 --> 00:11:01.908

so this is your final answer.

00:11:01.910 --> 00:11:04.250

It's how that original problem

00:11:04.250 --> 00:11:06.122

we were given factors,

00:11:06.130 --> 00:11:08.734

so it's just that bottoms up method

00:11:08.734 --> 00:11:10.809

makes it pretty easy to use.

00:11:10.810 --> 00:11:12.385
The only thing is you gotta remember

00:11:12.385 --> 00:11:13.676
if there's something to pull out

00:11:13.676 --> 00:11:15.034
at the beginning, like we had here.

00:11:15.034 --> 00:11:16.491
The three you gotta pull it out

00:11:16.491 --> 00:11:17.178
at the beginning,

00:11:17.180 --> 00:11:19.256
otherwise it's not going to work,

00:11:19.260 --> 00:11:20.512
but hopefully that helps

00:11:20.512 --> 00:11:22.077
you factor stuff a little.

00:11:22.080 --> 00:11:24.257
Easier and it works every single time,

00:11:24.260 --> 00:11:25.639

which is super nice and why my

00:11:25.639 --> 00:11:26.909

students tend to really like it.