00:00:07.320 --> 00:00:10.668
Right this video is an overview

00:00:10.668 --> 00:00:14.279
of like common number systems and.

00:00:14.280 --> 00:00:16.296
So like the number of systems that you

00:00:16.296 --> 00:00:18.456
might come across and use in math classes,

00:00:18.460 --> 00:00:20.560
and the one that people usually

00:00:20.560 --> 00:00:22.890
come across first is the the

00:00:22.890 --> 00:00:24.598
collection of natural numbers.

00:00:28.420 --> 00:00:30.262
And natural numbers.

00:00:30.262 --> 00:00:32.718
Actually it's not completely.

00:00:32.720 --> 00:00:33.682

Agreed upon.

00:00:33.682 --> 00:00:37.049
If zero is one of these numbers.

00:00:37.050 --> 00:00:39.142
I'm going to say it is so these

00:00:39.142 --> 00:00:40.200
are numbers like this, like

00:00:42.420 --> 00:00:44.492
0123 and so on, so they're like

00:00:44.492 --> 00:00:46.190
the counting numbers, right and?

00:00:46.190 --> 00:00:49.430
Every one of these number systems has a

00:00:49.518 --> 00:00:53.166
symbol like like a letter that represents it,

00:00:53.170 --> 00:00:55.490
so they're typically written in

00:00:55.490 --> 00:00:57.346
what's called Blackboard bold,

00:00:57.350 --> 00:00:59.150
and this is blackboard bold,

00:00:59.150 --> 00:01:00.442
like a capital N,

00:01:00.442 --> 00:01:02.670
and you put this extra line in.

00:01:02.670 --> 00:01:05.712
That's it, so this is the natural numbers and

00:01:05.712 --> 00:01:07.988
everything is fine in the natural numbers.

00:01:07.990 --> 00:01:10.132
As long as you're multiplying or

00:01:10.132 --> 00:01:11.870
adding right, like 3 + 2 is 5.

00:01:11.870 --> 00:01:13.122
That's another natural number,

00:01:13.122 --> 00:01:15.052
or like 3 * 2 or 6.

00:01:15.052 --> 00:01:15.948
That's another natural number.

00:01:15.950 --> 00:01:17.526
We're not going to run into any problems,

00:01:17.530 --> 00:01:19.525
as long as we're multiplying and adding.

00:01:19.530 --> 00:01:21.322
But if we want to subtract we

00:01:21.322 --> 00:01:23.029
we will run into problems,

00:01:23.030 --> 00:01:24.806
and so we need to include

00:01:24.810 --> 00:01:28.198
now negative numbers and.

00:01:28.200 --> 00:01:30.144
If we include the natural numbers

00:01:30.144 --> 00:01:31.810
and then all they're like.

00:01:31.810 --> 00:01:33.170
They're opposites, they're additive,

00:01:33.170 --> 00:01:33.850

inverse right?

00:01:33.850 --> 00:01:36.706

All the negatives of the natural numbers.

00:01:36.710 --> 00:01:38.117

Then we get what are called integers.

00:01:42.490 --> 00:01:44.160

The symbol for integers is.

00:01:46.290 --> 00:01:49.482

Blackboard Bold Z and that's because in

00:01:49.482 --> 00:01:52.677

German the word for numbers is zolan,

00:01:52.680 --> 00:01:55.188

so this is really just German

00:01:55.190 --> 00:01:57.580

numbers right? So and then.

00:01:57.580 --> 00:01:58.924

It's going to have all the negatives.

00:02:04.140 --> 00:02:06.373

OK, and now everything is fine again

00:02:06.373 --> 00:02:09.636
as long as we are adding or multiplying

00:02:09.636 --> 00:02:11.420
or subtracting right addition,

00:02:11.420 --> 00:02:13.740
multiplication, subtraction.

00:02:13.740 --> 00:02:15.852
All that stuff is fine unless we want

00:02:15.852 --> 00:02:18.492
to divide. So if we have like 5 / 2,

00:02:18.492 --> 00:02:23.030
we're not going to get an integer, so.

00:02:23.030 --> 00:02:24.810
I really want to put this symbol in between

00:02:24.852 --> 00:02:26.990
them, so I'm going to do it. This is.

00:02:30.230 --> 00:02:33.042
This means. Is contained in,

00:02:33.042 --> 00:02:34.662
so the natural numbers are

00:02:34.662 --> 00:02:36.396
contained in the integers, right?

00:02:36.396 --> 00:02:38.524
Every one of these is one of these,

00:02:38.530 --> 00:02:39.578
but not vice versa.

00:02:39.578 --> 00:02:41.150
So these guys then we can

00:02:41.218 --> 00:02:43.180
expand them out to what are

00:02:43.180 --> 00:02:44.488
called the rational numbers.

00:02:49.450 --> 00:02:51.250
I'm just going to put rationales.

00:02:51.250 --> 00:02:52.850
And people for rationals.

00:02:52.850 --> 00:02:55.362
They write blackboard bold Q like

00:02:55.362 --> 00:02:58.570

this Q is for quotient I guess.

00:02:58.570 --> 00:03:01.903
And this is any number that

00:03:01.903 --> 00:03:04.868
is a ratio of integers.

00:03:04.870 --> 00:03:09.310
So numbers like this like 2/3.

00:03:09.310 --> 00:03:12.910
3/5 negative 8. You know?

00:03:12.910 --> 00:03:17.870
9th Write 2 is one of these

00:03:17.870 --> 00:03:20.078
numbers because that's 2 / 1 right?

00:03:20.080 --> 00:03:23.116
So again, you have this containment.

00:03:23.120 --> 00:03:25.206
So every integer is a rational number,

00:03:25.210 --> 00:03:28.292
but not vice versa. Uhm?

00:03:28.292 --> 00:03:30.300
Here, OK, now here.

00:03:30.300 --> 00:03:32.310
Everything is great as long as

00:03:32.310 --> 00:03:33.482
we're multiplying, dividing,

00:03:33.482 --> 00:03:34.988
adding or subtracting,

00:03:34.988 --> 00:03:38.690
we're going to stay with rational numbers.

00:03:38.690 --> 00:03:39.554
So rational numbers.

00:03:39.554 --> 00:03:40.994
Actually, they're an example of

00:03:40.994 --> 00:03:42.447
what's called a field of numbers.

00:03:42.450 --> 00:03:43.560
It's just a collection of

00:03:43.560 --> 00:03:44.670
numbers where you can add,

00:03:44.670 --> 00:03:46.222
subtract, multiply and divide.

00:03:46.222 --> 00:03:47.386
That's basically it.

00:03:47.390 --> 00:03:48.374
Integers you can't divide,

00:03:48.374 --> 00:03:49.850
but you can do everything else.

00:03:49.850 --> 00:03:51.746
It's called a ring of numbers.

00:03:51.750 --> 00:03:54.160
It doesn't really matter so.

00:03:54.160 --> 00:03:56.182
Those are kind of the main

00:03:56.182 --> 00:03:57.530
number systems and then.

00:03:57.530 --> 00:03:59.210
What we might wanna do is

00:03:59.210 --> 00:04:01.020

what if we want to solve?

00:04:04.700 --> 00:04:08.102

You know? X ^2 = 2 and why would we

00:04:08.102 --> 00:04:09.845

do that? So like a really natural

00:04:09.845 --> 00:04:11.320

reason to do that actually is.

00:04:13.430 --> 00:04:16.270

Is this? Let's say I have a square.

00:04:16.270 --> 00:04:19.650

And this side is 1 and this side is 1.

00:04:19.650 --> 00:04:23.980

How long is the diagonal there well?

00:04:27.940 --> 00:04:31.028

Right, if we call the diagonal here X.

00:04:31.030 --> 00:04:32.790

Then by Pythagorean theorem right?

00:04:32.790 --> 00:04:34.020

1 ^2 + 1 squared,

00:04:34.020 --> 00:04:35.982
which is 2 is going to equal X squared.

00:04:35.990 --> 00:04:39.040
So we need to solve this equation and

00:04:39.040 --> 00:04:43.138
around 2500 years ago people showed that.

00:04:43.140 --> 00:04:45.092
None of those numbers.

00:04:45.092 --> 00:04:46.556
Have this property,

00:04:46.560 --> 00:04:48.288
none of them solve this equation,

00:04:48.290 --> 00:04:50.514
so we need to expand out from the

00:04:50.514 --> 00:04:52.460
rationals into a bigger number system,

00:04:52.460 --> 00:04:54.740
and there are several ways now

00:04:54.740 --> 00:04:57.119
that that that can happen so.

00:04:57.120 --> 00:04:59.188

But usually you know.

00:04:59.188 --> 00:05:02.680

Just talk about the real numbers so.

00:05:04.690 --> 00:05:06.040

Real numbers are written like this.

00:05:08.650 --> 00:05:10.666

And OK, how can we like say,

00:05:10.670 --> 00:05:12.030

what the real numbers are?

00:05:12.030 --> 00:05:14.340

They're just numbers with decimals,

00:05:14.340 --> 00:05:15.690

infinitely many decimals,

00:05:15.690 --> 00:05:20.280

so real numbers include, like sqrt 2.

00:05:20.280 --> 00:05:21.798

Sqrt 2 has a decimal right?

00:05:21.800 --> 00:05:23.326

It's like you can get a calculator.

00:05:23.330 --> 00:05:24.324

You can see a bunch of it,

00:05:24.330 --> 00:05:25.760

but it goes on forever,

00:05:25.760 --> 00:05:29.144

so it's 1.414 blah blah

00:05:29.144 --> 00:05:30.934

blah blah blah blah blah.

00:05:30.940 --> 00:05:32.990

Right, or they're they're all

00:05:32.990 --> 00:05:35.040

these other numbers like π.

00:05:35.040 --> 00:05:37.250

Which is 3.141.

00:05:39.580 --> 00:05:41.176

59 blah blah blah blah right.

00:05:41.180 --> 00:05:43.850

And these decimals repeat forever.

00:05:43.850 --> 00:05:45.761

And they're going to be negative numbers

00:05:45.761 --> 00:05:48.330

like negative 8 is a real number, and so on.

00:05:48.330 --> 00:05:50.070

All the rational numbers also have

00:05:50.070 --> 00:05:52.913

these decimal expansions, so you know.

00:05:52.913 --> 00:05:54.797

So the rational numbers.

00:05:54.800 --> 00:05:55.811

Let's see here.

00:05:55.811 --> 00:05:57.159

I'm behind the symbols,

00:05:57.160 --> 00:05:58.200

but I think it's going to be OK,

00:05:58.200 --> 00:06:02.280

so they have infinite decimal expansions,

00:06:02.280 --> 00:06:03.906

but they repeat.

00:06:03.906 --> 00:06:07.158

So if you think about 2/3.

00:06:07.160 --> 00:06:07.510

2/3 is

00:06:09.900 --> 00:06:13.430

0.6666 right? It just goes on forever or 3/5.

00:06:18.090 --> 00:06:22.454

0.60 The Zeros just go on forever, right?

00:06:22.454 --> 00:06:25.410

So eventually the decimals for

00:06:25.410 --> 00:06:27.570

rational numbers start repeating,

00:06:27.570 --> 00:06:29.929

and that's that's if and only if.

00:06:29.930 --> 00:06:31.610

So, that's if you have a number

00:06:31.610 --> 00:06:33.455

whose decimal start repeating

00:06:33.455 --> 00:06:36.130

its rational and vice versa.

00:06:36.130 --> 00:06:37.243

So over here.

00:06:37.243 --> 00:06:39.840

There are all these numbers where the

00:06:39.915 --> 00:06:43.184

decimals never fall into a repeating pattern.

00:06:43.190 --> 00:06:44.306

They might fall into a pattern,

00:06:44.310 --> 00:06:46.137

but it's not 1 where pieces of

00:06:46.137 --> 00:06:48.028

it repeat over and over and over,

00:06:48.030 --> 00:06:49.590

and those numbers are called irrational.

00:06:49.590 --> 00:06:51.950

They're just real numbers that

00:06:51.950 --> 00:06:54.490

aren't rational. And then OK,

00:06:54.490 --> 00:06:56.735

so every rational number is a real number.

00:06:56.740 --> 00:06:59.698

So this kind of chain continues.

00:06:59.700 --> 00:07:03.480

If we want to solve equations like X,

00:07:03.480 --> 00:07:05.900

squared is minus one.

00:07:05.900 --> 00:07:07.620

No real number is going to do that, right?

00:07:07.620 --> 00:07:09.580

If you square one of these numbers,

00:07:09.580 --> 00:07:12.170

you're going to get zero or bigger, right?

00:07:12.170 --> 00:07:13.410

You can't get negative

00:07:13.410 --> 00:07:14.650

numbers out of squaring.

00:07:14.650 --> 00:07:16.378

So then we can include what

00:07:16.378 --> 00:07:17.530

are called complex numbers,

00:07:17.530 --> 00:07:19.414

so complex numbers.

00:07:19.414 --> 00:07:21.298

Are things like.

00:07:21.300 --> 00:07:26.117

Two plus and here I squared is minus one.

00:07:26.120 --> 00:07:28.290

It's the square root of negative one.

00:07:28.290 --> 00:07:30.054

So complex numbers are written like this.

00:07:36.000 --> 00:07:37.458

All real numbers are complex numbers.

00:07:37.460 --> 00:07:39.236

They just don't have the plus I write.

00:07:39.240 --> 00:07:40.155

So let's let's just write

00:07:40.155 --> 00:07:41.220

another one of these like so.

00:07:41.220 --> 00:07:44.734

For example, we can have negative 3.

00:07:44.740 --> 00:07:48.328

Plus five, these are complex numbers.

00:07:48.330 --> 00:07:49.730

And these are really, really,

00:07:49.730 --> 00:07:51.425

really, really useful.

00:07:51.425 --> 00:07:53.685

It's it's sort of.

00:07:53.690 --> 00:07:54.915

It's sort of this misunderstanding

00:07:54.915 --> 00:07:56.761

that this is sort of like a thing

00:07:56.761 --> 00:07:57.907

that you do in algebra just

00:07:57.907 --> 00:07:59.308

to solve quadratic equations,

00:07:59.310 --> 00:08:00.678
but it's not.

00:08:00.678 --> 00:08:03.870
It's really way more than that so.

00:08:03.870 --> 00:08:04.860
For example.

00:08:07.550 --> 00:08:09.746
There's a thing called fast Fourier

00:08:09.746 --> 00:08:11.732
transform and this fast Fourier

00:08:11.732 --> 00:08:14.007
transform relies on complex numbers,

00:08:14.010 --> 00:08:16.558
and your phone is using things that

00:08:16.558 --> 00:08:19.370
are at least very much like fast

00:08:19.370 --> 00:08:21.830
Fourier transform so that you can,

00:08:21.830 --> 00:08:24.370
like you know, stream video.

00:08:24.370 --> 00:08:26.455
So whenever you're doing like

00:08:26.455 --> 00:08:28.540
streaming video or streaming audio.

00:08:28.540 --> 00:08:30.170
All kinds of complex numbers

00:08:30.170 --> 00:08:31.800
are making that work right?

00:08:31.800 --> 00:08:34.680
Or just recently there was a

00:08:34.680 --> 00:08:37.851
paper published in in this journal

00:08:37.851 --> 00:08:40.838
called Nature that was essentially

00:08:40.838 --> 00:08:44.430
arguing that complex numbers maybe.

00:08:46.960 --> 00:08:50.428
Not only an inherent part of.

00:08:50.430 --> 00:08:51.448

Quantum mechanics.

00:08:51.448 --> 00:08:53.993
But but that that hypothesis

00:08:53.993 --> 00:08:56.594
are complex numbers inherent to

00:08:56.594 --> 00:08:59.089
quantum mechanics may actually be,

00:08:59.090 --> 00:09:00.562
you know, scientifically testable

00:09:00.562 --> 00:09:02.770
that you could design an experiment

00:09:02.827 --> 00:09:04.395
for that so that these are like

00:09:04.395 --> 00:09:06.028
as real as the real numbers.

00:09:06.030 --> 00:09:07.590
It's just for historical reasons.

00:09:07.590 --> 00:09:08.772
They have parts that are called

00:09:08.772 --> 00:09:09.830

imaginary and so on there.

00:09:09.830 --> 00:09:10.460

So that's it.

00:09:10.460 --> 00:09:11.720

Let me say one more thing,

00:09:11.720 --> 00:09:13.508

and then then we'll quit here

00:09:13.510 --> 00:09:15.810

in between the rational numbers

00:09:15.810 --> 00:09:17.458

and the real numbers.

00:09:17.458 --> 00:09:19.106

There's another number field.

00:09:22.050 --> 00:09:23.718

Which is called the algebraic numbers.

00:09:28.870 --> 00:09:31.810

And algebraic numbers are there.

00:09:31.810 --> 00:09:33.575

The solutions of polynomial equations

00:09:33.575 --> 00:09:35.738

where all the numbers in the

00:09:35.738 --> 00:09:38.310

equation are integers, like this guy.

00:09:38.310 --> 00:09:41.370

So sqrt 2 is over here.

00:09:41.370 --> 00:09:42.990

Which makes it a real number,

00:09:42.990 --> 00:09:46.506

but Π is not in here,

00:09:46.510 --> 00:09:49.006

and this problem was known to the Greeks

00:09:49.006 --> 00:09:51.377

and another problem known to the Greeks.

00:09:51.380 --> 00:09:53.984

This figure is getting too long was.

00:09:53.990 --> 00:09:57.644

Can you can you take a circle?

00:09:57.650 --> 00:09:59.518

And through rural encompass,

00:09:59.518 --> 00:10:02.710
construct a square with the same area

00:10:02.710 --> 00:10:05.790
and that problem stood open for like 2000

00:10:05.790 --> 00:10:08.349
years before it was eventually shown.

00:10:08.350 --> 00:10:11.461
No, you can't do that and that no,

00:10:11.461 --> 00:10:13.129
you can't do that,

00:10:13.130 --> 00:10:15.046
which is a geometry thing, right?

00:10:15.046 --> 00:10:19.800
Really, amounts to Π is.

00:10:19.800 --> 00:10:22.980
A non algebraic real number.

00:10:22.980 --> 00:10:24.236
So they're just numbers.

00:10:24.236 --> 00:10:25.178

Are you know?

00:10:25.180 --> 00:10:26.220

Of course, they're super useful,

00:10:26.220 --> 00:10:28.088

but there's all this.

00:10:28.090 --> 00:10:29.700

Right there all this deep

00:10:29.700 --> 00:10:31.310

interesting stuff connected to them.

00:10:31.310 --> 00:10:31.988

So that's it.

00:10:31.988 --> 00:10:33.118

That's sort of an overview

00:10:33.118 --> 00:10:34.300

of common number systems.